

THE UNITY OF GEOMETRY

JOSÉ G. VARGAS

Geometry evolved in a haphazard way until the mid nineteenth century. Key to the transition to a more structured phase were Cayley, Grassmann and Felix Klein. Cartan put their new ideas together and, after the pertinent corrections, he generalized it. But he only partially tried to explain how all of this fits together. Using his ideas, we shall

(a) get very naturally to the concept of frame bundle by addressing the issue of the matching of an affine space, which does not have a special point, with a vector space, which has a special element, the zero;

(b) explain Cartan's correction and generalization of the Erlangen program,

(c) show how (b) leads to his Clifford connections (ignored in the literature), Finslerian bundles, canonical Kaluza-Klein space (This is Oscar Klein) and a better understanding of projective geometry.